

(1+1)-dimensional gauge symmetric gravity model and related exact black hole and cosmological solutions in string theory

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Abstract

We introduce a four-dimensional extension of the Poincaré algebra (\mathcal{N}) in $(1+1)$ -dimensional space-time and obtain a $(1+1)$ -dimensional gauge symmetric gravity model using the algebra \mathcal{N} . We show that the obtained gravity model is dual (canonically transformed) to the $(1+1)$ -dimensional *AdS* gravity. We also obtain some black hole and Friedmann-Robertson-Walker (FRW) solutions by solving its classical equations of motion. Then, we study $[SL(2, \mathbb{R}) \times \mathbf{R}]/[U(1) \times U(1)]$ gauged Wess-Zumino-Witten (WZW) model and obtain some exact black hole and cosmological solutions in string theory. We show that some obtained black hole and cosmological metrics in string theory are same as the metrics obtained in solutions of our gauge symmetric gravity model.

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1 Introduction

(1 + 1)-dimensional gravity has been extensively studied by proposing various models. Two of that gravities has been most interested because of both their simplicity and their group theoretical properties. One of them is proposed by Jackiw [1] and Teitelboim [2] (Liouville gravity) which is equivalent to the gauge theory of gravity with (anti) de sitter group [3, 4, 5]. The other one is the string-inspired gravity [6, 7, 33] which is equivalent to the gauge theory of the Poincaré group $ISO(1, 1)$ [7] and it's central extension [9, 10, 11, 12, 13].

Recently, two algebras namely the Maxwell algebra [14, 15] and the semi-simple extension of the Poincaré algebra [16] have been applied to construct some gauge invariant theories of gravity in four [16, 17, 18, 19, 20] and three [21, 22, 23] dimensional space-times. These algebras have been also applied to string theory as an internal symmetry of the matter gauge fields [24]. The Maxwell algebra in (1 + 1)-dimensional space-time, is the well-known central extension of the Poincaré algebra which as we discussed above, has been applied to construct a (1 + 1)-dimensional gauge symmetric gravity action [9, 10]. In this paper, we introduce a new four-dimensional extension of the Poincaré algebra (\mathcal{N}) in (1 + 1)-dimensional space-time, which obtains from the 16-dimensional semi-simple extension of Poincaré algebra in (3 + 1)-dimensional space-time [16], by reduction of dimensions of the space. Then, we construct a (1 + 1)-dimensional gauge symmetric gravity model, using this algebra. We obtain some black hole and cosmological solutions by solving it's equations of motion.

On the other hand, in string theory, two-dimensional exact black hole has been found by Witten [6]. Another black hole solution to the string theory has been presented in [25] both in Schwarzschild-like and target space conformal gauges. Exact three-dimensional black string and black hole solutions in string theory are also have been found in [26, 27]. Here, we study the string theory in (1 + 1)-dimensional space-time, and show that some obtained black hole and cosmological solutions of the gravity model, are exact solutions of the beta function equations (in all loops).

The outlines of this paper is as follows: In section two, we construct a (1 + 1)-dimensional gauge symmetric gravity model using a four-dimensional gauge group related to the algebra \mathcal{N} . Then, by presenting a canonical map, we show that the obtained gravity model is dual (canonically transformed) to the (1 + 1)-dimensional *Ads* gravity model. In section three, we solve the equations of motion and obtain some black hole and Friedmann-Robertson-Walker (FRW) cosmological solutions. Finally, in section four, we study $[SL(2, \mathbb{R}) \times \mathbf{R}]/[U(1) \times U(1)]$ gauged Wess-Zumino-Witten (WZW) model, and show that some of the resulting string backgrounds, which are exact (1 + 1)-dimensional solutions of the string theory, are same as the black hole and cosmological solutions obtained for our gravity model. Section five, contains some concluding remarks.

2 (1 + 1)-dimensional gravity from a non-semi-simple extension of the Poincaré gauge symmetric model

The Poincaré algebra $ISO(1, 1)$ in (1 + 1)-dimensional space-time has the following form:

$$[J, P_a] = \epsilon_{ab} P^b, \quad [P_a, P_b] = 0, \quad (1)$$

where $\epsilon^{01} = -\epsilon_{01} = +1$, and J and P_a ($a = 0, 1$) are generators of the rotation and translations in space-time. In $D = 1 + 1$, a four-dimensional non-semi-simple extension of the Poincaré algebra¹ $\mathcal{N} = (P_a, J, Z)$ has the following form:

$$[J, P_a] = \epsilon_{ab} P^b, \quad [P_a, P_b] = k \epsilon_{ab} Z, \quad [Z, P_a] = -\frac{\Lambda}{k} \epsilon_{ab} P^b, \quad (2)$$

where Z is the new generator and k and Λ are constants.² For $\Lambda = 0$, which leads to $[Z, P_a] = 0$, the above algebra reduces to a solvable algebra which is called the centrally extended Poincaré algebra (or Maxwell algebra³ in 1 + 1 dimensions) [9, 10, 11]. We construct the \mathcal{N} -algebra valued one-form gauge field as follows:

$$h_i = h_i^B X_B = e_i^a P_a + \omega_i J + A_i Z, \quad i, j = 0, 1 \quad (3)$$

where the indices $i, j = 0, 1$ are the space-time indices, and the one-form fields have the following forms:

$$e^a = e_i^a dx^i, \quad \omega = \omega_i dx^i, \quad A = A_i dx^i,$$

¹This algebra is isomorphic to the four-dimensional Lie algebra $A_{3,8} \oplus A_1$.

²Note that the commutator $[J, Z] = 0$ can be obtained from Jacobi identity.

³Centrally extended Poincaré algebra (or Maxwell algebra) in 1 + 1 dimensions is isomorphic to the four-dimensional Lie algebra $A_{4,8}$ [28].

where e_i^a, ω_i, A_i are the vierbein, spin connection and the new gauge field, respectively. Using the following infinitesimal gauge parameter:

$$u = \rho^a P_a + \tau J + \lambda Z,$$

and the gauge transformation as follows:

$$h_i \rightarrow h'_i = U^{-1} h_i U + U^{-1} \partial_i U,$$

with $U = e^{-u} \simeq 1 - u$ and $U^{-1} = e^u \simeq 1 + u$, we obtain the following transformations of the gauge fields:

$$\begin{aligned} \delta e_i^a &= -\partial_i \rho^a - \epsilon^{ab} e_{ib} \left(\tau - \frac{\Lambda}{k} \lambda \right) + \epsilon^{ab} \rho_b \left(\omega_i - \frac{\Lambda}{k} A_i \right), \\ \delta \omega_i &= -\partial_i \tau, \\ \delta A_i &= -\partial_i \lambda - k \epsilon^{ab} e_{ia} \rho_b. \end{aligned} \quad (4)$$

The generic Ricci curvature can be obtained as follows:

$$\begin{aligned} \mathcal{R} &= \mathcal{R}_{ij} dx^i \wedge dx^j = \mathcal{R}^A X_A = \mathcal{R}_{ij}^A X_A dx^i \wedge dx^j, \\ \mathcal{R}_{ij} &= \partial_{[i} h_{j]} + [h_i, h_j] = \mathcal{R}_{ij}^A X_A = T_{ij}^a P_a + R_{ij} J + F_{ij} Z, \end{aligned} \quad (5)$$

where the torsion T_{ij}^a , standard Riemannian curvature R_{ij} and the new gauge field strength F_{ij} have the following forms:

$$\begin{aligned} T_{ij}^a &= \partial_{[i} e_{j]}^a + \epsilon^{ab} (e_{ib} \omega_j - e_{jb} \omega_i) - \frac{\Lambda}{k} \epsilon^{ab} (e_{ib} A_j - e_{jb} A_i), \\ R_{ij} &= \partial_{[i} \omega_{j]}, \\ F_{ij} &= \partial_{[i} A_{j]} + k \epsilon^{ab} e_{ia} e_{jb}. \end{aligned} \quad (6)$$

Now, one can write the gauge invariant action as

$$I = \frac{1}{2} \int \eta_A \mathcal{R}^A = \frac{1}{2} \int d^2 x \epsilon^{ij} \eta_A \mathcal{R}_{ij}^A \quad (7)$$

$$= \frac{1}{2} \int d^2 x \epsilon^{ij} (\eta_a T_{ij}^a + \eta_2 R_{ij} + \eta_3 F_{ij}), \quad (8)$$

where $\eta_A = (\eta_a, \eta_2, \eta_3)$ are the lagrange multiplier-like fields. Now, using (6), one can rewrite this action in the following form:

$$I = \int d^2 x \epsilon^{ij} \left\{ \eta_a \left(\partial_i e_j^a + \epsilon^{ab} e_{ib} \left(\omega_j - \frac{\Lambda}{k} A_j \right) \right) + \eta_2 \partial_i \omega_j + \eta_3 \left(\partial_i A_j + \frac{1}{2} k \epsilon^{ab} e_{ia} e_{jb} \right) \right\}. \quad (9)$$

This action is invariant under the gauge transformations (4) and the following transformations of the fields η_a, η_2 and η_3 :

$$\begin{aligned} \delta \eta_a &= k \epsilon_a^b \eta_3 \rho_b - \epsilon_a^b \eta_b \left(\tau - \frac{\Lambda}{k} \lambda \right), \\ \delta \eta_2 &= -\epsilon^{ab} \eta_a \rho_b, \\ \delta \eta_3 &= \frac{\Lambda}{k} \epsilon^{ab} \eta_a \rho_b. \end{aligned} \quad (10)$$

Now, we will show that this model is dual to the (1+1)-dimensional *Ads* gravity. We know that $SO(2, 1)$ gauge symmetric gravity action can be obtained by use of the following algebra (anti de Sitter algebra for $k' \neq 0$):

$$[J, P_a] = \epsilon_{ab} P^b, \quad [P_a, P_b] = k' \epsilon_{ab} J, \quad (11)$$

as follows: [10]

$$\tilde{I} = \int d^2 x \epsilon^{ij} \left\{ \tilde{\eta}_a \left(\partial_i e_j^a + \epsilon^{ab} e_{ib} \omega_j \right) + \tilde{\eta}_2 \left(\partial_i \omega_j + \frac{1}{2} k' \epsilon^{ab} e_{ia} e_{jb} \right) \right\}. \quad (12)$$

Indeed, by selecting $\eta_3 = -\frac{\Lambda}{k}\eta_2$ in our model (9), it is dual to the *Ads* gravity (12); i.e. the following map:

$$\omega_i \longrightarrow \omega_i - \frac{\Lambda}{k}A_i, \quad e_i^a \longrightarrow e_i^a, \quad \tilde{\eta}_a \longrightarrow \eta_a, \quad \tilde{\eta}_2 \longrightarrow \eta_2, \quad k' = -\Lambda, \quad (13)$$

transforms the *Ads*₂ gravity model (12) to our model (9). In the following, we will show that this map is a canonical one. The canonical Poisson-brackets and the Hamiltonian related to the *Ads*₂ gravity model (12) are as follows:

$$\begin{aligned} \{(\tilde{\Pi}_e)_i^a(x), e_j^b(y)\} &= \epsilon_{ij}\eta^{ab}\delta(x-y), & \{(\tilde{\Pi}_\omega)_i(x), \omega_j(y)\} &= \epsilon_{ij}\delta(x-y), \\ \{(\tilde{\Pi}_{\tilde{\eta}_a})^a(x), \tilde{\eta}^b(y)\} &= \eta^{ab}\delta(x-y), & \{(\tilde{\Pi}_{\tilde{\eta}_2})(x), \tilde{\eta}_2(y)\} &= \delta(x-y), \\ \tilde{H} &= \int d^3x \left((\tilde{\Pi}_e)_a^i \partial_t e_i^a + (\tilde{\Pi}_\omega)_a^i \partial_t \omega_i^a + (\tilde{\Pi}_{\tilde{\eta}_a})^a \partial_t \tilde{\eta}_a + (\tilde{\Pi}_{\tilde{\eta}_2}) \partial_t \tilde{\eta}_2 \right) - \tilde{I} \\ &= 2 \int d^2x \epsilon^{0i} \left(\tilde{\eta}_a \partial_t e_i^a + \tilde{\eta}_2 \partial_t \omega_i \right) - \tilde{I}, \end{aligned} \quad (14)$$

where η^{ab} is the inverse Minkowski metric, and the conjugate momentums corresponding to the fields are as follows:

$$\begin{aligned} (\tilde{\Pi}_e)_a^i &= \frac{\partial \tilde{\mathcal{L}}}{\partial(\partial_t e_i^a)} = \epsilon^{0i} \tilde{\eta}_a, & (\tilde{\Pi}_\omega)^i &= \frac{\partial \tilde{\mathcal{L}}}{\partial(\partial_t \omega_i)} = \epsilon^{0i} \tilde{\eta}_2, \\ (\tilde{\Pi}_{\tilde{\eta}_a})^a &= \frac{\partial \tilde{\mathcal{L}}}{\partial(\partial_t \tilde{\eta}_a)} = -\epsilon^{0i} e_i^a, & (\tilde{\Pi}_{\tilde{\eta}_2}) &= \frac{\partial \tilde{\mathcal{L}}}{\partial(\partial_t \tilde{\eta}_2)} = -\epsilon^{0i} \omega_i. \end{aligned} \quad (15)$$

The map (13) is a canonical transformation and easily it can be shown that, under this map, the canonical Poisson-brackets and the Hamiltonian (14) related to the *Ads*₂ gravity model (12) are transformed to the following Poisson-brackets and the Hamiltonian related to our model (9):

$$\begin{aligned} \{(\Pi_e)_i^a(x), e_j^b(y)\} &= \epsilon_{ij}\eta^{ab}\delta(x-y), & \{(\Pi_\omega)_i(x), \omega_j(y)\} &= \epsilon_{ij}\delta(x-y), \\ \{(\Pi_A)_i(x), A_j(y)\} &= \epsilon_{ij}\delta(x-y), & \{(\Pi_{\eta_a})^a(x), \eta^b(y)\} &= \eta^{ab}\delta(x-y), \\ \{(\Pi_{\eta_2})(x), \eta_2(y)\} &= \delta(x-y), & \{(\Pi_{\eta_3})(x), \eta_3(y)\} &= \delta(x-y), \\ H &= \int d^3x \left((\Pi_e)_a^i \partial_t e_i^a + (\Pi_\omega)_a^i \partial_t \omega_i^a + (\Pi_A)_a^i \partial_t A_i^a + (\Pi_{\eta_a})^a \partial_t \eta_a + (\Pi_{\eta_2}) \partial_t \eta_2 + (\Pi_{\eta_3}) \partial_t \eta_3 \right) - I \\ &= 2 \int d^2x \epsilon^{0i} \left(\eta_a \partial_t e_i^a + \eta_2 \partial_t \omega_i + \eta_3 \partial_t A_i \right) - I, \end{aligned} \quad (16)$$

where the conjugate momentums corresponding to the fields are given as follows:

$$\begin{aligned} (\Pi_e)_a^i &= \frac{\partial \mathcal{L}}{\partial(\partial_t e_i^a)} = \epsilon^{0i} \eta_a, & (\Pi_\omega)^i &= \frac{\partial \mathcal{L}}{\partial(\partial_t \omega_i)} = \epsilon^{0i} \eta_2, \\ (\Pi_A)^i &= \frac{\partial \mathcal{L}}{\partial(\partial_t A_i)} = \epsilon^{0i} \eta_3, & (\Pi_{\eta_a})^a &= \frac{\partial \mathcal{L}}{\partial(\partial_t \eta_a)} = -\epsilon^{0i} e_i^a, \\ (\Pi_{\eta_2}) &= \frac{\partial \mathcal{L}}{\partial(\partial_t \eta_2)} = -\epsilon^{0i} \omega_i, & (\Pi_{\eta_3}) &= \frac{\partial \mathcal{L}}{\partial(\partial_t \eta_3)} = -\epsilon^{0i} A_i. \end{aligned} \quad (17)$$

Note that the conjugate momentums are transformed under the map (13) as:

$$\begin{aligned} (\tilde{\Pi}_e)_a^i &\longrightarrow (\Pi_e)_a^i, & (\tilde{\Pi}_{\tilde{\eta}_a})^a &\longrightarrow (\Pi_{\eta_a})^a, \\ (\tilde{\Pi}_\omega)^i &\longrightarrow (\Pi_\omega)^i, & \tilde{\Pi}_{\tilde{\eta}_2} &\longrightarrow (\Pi_{\eta_2} - \frac{\Lambda}{k} \Pi_{\eta_3}). \end{aligned}$$

Since the Poisson-brackets and Hamiltonian of the model are preserved under the map (13), then the *Ads* gravity model (12) is dual to our gravity model (9), and each can be transformed to the other by the canonical transformation (13), of course with $\eta_3 = -\frac{\Lambda}{k}\eta_2$. Finally, under the map (13), the equations of motion for the *Ads*₂ gravity (12) also transform to the equations of motion related to our model (9).

The equations of motion with respect to the fields η_a, η_2, η_3 have the following forms, respectively:

$$\begin{aligned}\epsilon^{ij} \left(\partial_i e_j^a + \epsilon^{ab} e_{ib} \left(\omega_j - \frac{\Lambda}{k} A_j \right) \right) &= 0, \\ \epsilon^{ij} \partial_i \omega_j &= 0, \\ \epsilon^{ij} \left(\partial_i A_j + \frac{1}{2} k \epsilon^{ab} e_{ia} e_{jb} \right) &= 0,\end{aligned}\tag{18}$$

and the equations of motion with respect to the fields e_i^a, ω_i, A_i are obtained as follows, respectively:

$$\begin{aligned}\epsilon^{ij} \left(-\partial_j \eta_a + \epsilon_a^b \eta_b \left(\omega_j - \frac{\Lambda}{k} A_j \right) - k \epsilon_{ab} \eta_3 e_j^b \right) &= 0, \\ \epsilon^{ij} \left(\partial_j \eta_2 - \epsilon^{ab} \eta_a e_{jb} \right) &= 0, \\ \epsilon^{ij} \left(\partial_j \eta_3 + \frac{\Lambda}{k} \epsilon^{ab} \eta_a e_{jb} \right) &= 0.\end{aligned}\tag{19}$$

In the next section, we will try to solve these equations and obtain different solutions of them.

3 Solutions of the equations of motion

3.1 Radial Solutions for $\Lambda \neq 0$

Using the following ansatz for the metric:

$$ds^2 = e_i^a e_j^b \eta_{ab} dx^i dx^j = -N^2(r) dt^2 + M^2(r) dr^2,\tag{20}$$

where $\{x^0, x^1\} = \{t, r\}$ are the coordinates of the space-time ($0 \leq t < \infty$, $-\infty < r < \infty$), one can obtain the following solution for the equations of motion (18)-(19):

$$\begin{aligned}M^2(r) &= \frac{1}{-\Lambda N^2(r) + C_4} \left(\frac{dN(r)}{dr} \right)^2, & \eta_0(r) &= C_2 N(r), & \eta_1(r) &= 0, \\ \eta_2(r) &= -\frac{C_2}{\Lambda} \sqrt{-\Lambda N^2(r) + C_4} + C_1, & \eta_3(r) &= \frac{C_2}{k} \sqrt{-\Lambda N^2(r) + C_4}, \\ \omega(r) &= C_3 dt + f(r) dr, & A(r) &= \frac{k}{\Lambda} \left((\sqrt{-\Lambda N^2(r) + C_4} + C_3) dt + f(r) dr \right),\end{aligned}\tag{21}$$

where C_1, C_2, C_3 and C_4 are arbitrary constants, and $N(r), f(r)$ are arbitrary functions of r . The solution (21) describes a space-time with a constant scalar curvature $\mathcal{R} = 2\Lambda$.

3.1.1 *Ads* black hole solution

For $N^2(r) = -\Lambda r^2 - b$ and $C_4 = -\Lambda b$, the solution (21) reduces to the following *Ads* black hole solution:

$$ds^2 = -N^2(r) dt^2 + \frac{dr^2}{N^2(r)},\tag{22}$$

and

$$\begin{aligned}\eta_0(r) &= C_2 N(r), & \eta_1(r) &= 0, \\ \eta_2(r) &= C_2 r + C_1, & \eta_3(r) &= -\frac{\Lambda}{k} C_2 r, \\ \omega(r) &= C_3 dt + f(r) dr, & A(r) &= (-kr + \frac{k}{\Lambda} C_3) dt + \frac{k}{\Lambda} f(r) dr,\end{aligned}\tag{23}$$

where b is a constant. Now, we calculate the mass (energy) of solution (23) using the ADM definition of mass (energy) as discussed in [29]. Varying the action (9) produces a bulk term, which is zero using the equations of motion, plus a boundary term which can be canceled by adding the following boundary term to the Lagrangian:

$$\mathcal{L}_B = -\partial_r \left(\eta_a e_0^a + \eta_2 \omega_0 + \eta_3 A_0 \right), \quad (24)$$

together with an appropriate boundary condition. This boundary term is identified as the mass (energy) of solution. Our boundary condition is using the obtained values for fields in the solution (23) at spatial infinity ($r \rightarrow \pm\infty$). Then, the mass of the solution is obtained as follows:

$$m = \int_{-\infty}^{+\infty} dr \mathcal{L}_B = - \left(\eta_a e_0^a + \eta_2 \omega_0 + \eta_3 A_0 \right) \Big|_{-\infty}^{+\infty}, \quad (25)$$

which using (22) and (23) turns out to be

$$m = C_2 b - C_1 C_3. \quad (26)$$

The Kretschmann scalar for this metric is

$$K = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 4\Lambda^2. \quad (27)$$

Consequently, this solution has two singularities at the following points:

$$r_{\pm} = \pm \sqrt{\frac{b}{-\Lambda}}, \quad (28)$$

which are not the curvature singularities, but the coordinate singularities, and can be removed by definition of a new coordinate system. Using the ansatz (22), we obtain another solution for the equations of motion (18)-(19) as follows:

$$\begin{aligned} N^2(r) &= -\Lambda r^2 - 2Dr + C_3, & \eta_0(r) &= C_2 N(r), & \eta_1(r) &= 0, \\ \eta_2(r) &= C_2 r + C_1, & \eta_3(r) &= -\frac{C_2}{k}(\Lambda r + D), \\ \omega(r) &= C_4 dt + f(r) dr, & A(r) &= (-kr + C_5) dt + \frac{k}{\Lambda} f(r) dr, \end{aligned} \quad (29)$$

where C_1, C_2, C_3, C_4, C_5 and $D = C_4 - \frac{\Lambda}{k} C_5$ are arbitrary constants, and $f(r)$ is a function of r . The value of the Kretschmann scalar for this solution is same as that of the previous one $K = 4\Lambda^2$, and it has two coordinate singularities at points

$$r_{\pm} = \frac{-D \pm \sqrt{D^2 + \Lambda C_3}}{\Lambda}. \quad (30)$$

Using new coordinate $\rho = r + \frac{D}{2\Lambda}$, the latter solution (29) transforms to the *Ads* black hole solution (23). This also can be achieved by choosing $D = 0$ and $C_3 = -b$.

3.1.2 Black hole solutions

For negative Λ , by assuming $N(r) = \sinh(\sqrt{-\Lambda} r - b)$ and $C_4 = -\Lambda$, the solution (21) reduces to the following black hole solution:

$$ds^2 = -\sinh^2(\sqrt{-\Lambda} r - b) dt^2 + dr^2, \quad (31)$$

$$\begin{aligned} \eta_0(r) &= C_2 \sinh(\sqrt{-\Lambda} r - b), & \eta_1(r) &= 0, \\ \eta_2(r) &= \frac{C_2}{\sqrt{-\Lambda}} \cosh(\sqrt{-\Lambda} r - b) + C_1, & \eta_3(r) &= \frac{C_2}{k} \sqrt{-\Lambda} \cosh(\sqrt{-\Lambda} r - b), \\ \omega(r) &= C_3 dt + f(r) dr, & A(r) &= \frac{k}{\Lambda} \left((\sqrt{-\Lambda} \cosh(\sqrt{-\Lambda} r - b) + C_3) dt + f(r) dr \right), \end{aligned} \quad (32)$$

where b is an arbitrary constant.

For positive Λ , by assuming $N(r) = \sin(\sqrt{\Lambda} r - b)$ and $C_4 = \Lambda$, the solution (21) reduces to the following black hole solution:

$$ds^2 = -\sin^2(\sqrt{\Lambda} r - b) dt^2 + dr^2, \quad (33)$$

$$\begin{aligned} \eta_0(r) &= C_2 \sin(\sqrt{\Lambda} r - b), & \eta_1(r) &= 0, \\ \eta_2(r) &= -\frac{C_2}{\sqrt{\Lambda}} \cos(\sqrt{\Lambda} r - b) + C_1, & \eta_3(r) &= \frac{C_2}{k} \sqrt{\Lambda} \cos(\sqrt{\Lambda} r - b), \\ \omega(r) &= C_3 dt + f(r) dr, & A(r) &= \frac{k}{\Lambda} \left((\sqrt{\Lambda} \cos(\sqrt{\Lambda} r - b) + C_3) dt + f(r) dr \right). \end{aligned} \quad (34)$$

Both of the solutions (31) and (33) have coordinate singularities at

$$r = \frac{b}{\sqrt{|\Lambda|}}, \quad (35)$$

which can be removed by suitable coordinate transformations, because of their finite Ricci and Kretschmann scalars.

3.2 Radial solutions for $\Lambda = 0$

As previously discussed, for $\Lambda = 0$, the non-semi-simple extension of the Poincaré algebra \mathcal{N} , reduces to the centrally extended Poincaré algebra. Then, the (1+1)-dimensional gauge symmetric gravity model (9) reduces to the following central extension of Poincaré gauge symmetric action [9, 10, 11]:

$$I = \int d^2x \epsilon^{ij} \left\{ \eta_a \left(\partial_i e_j^a + \epsilon^{ab} e_{ib} \omega_j \right) + \eta_2 \partial_i \omega_j + \eta_3 \left(\partial_i A_j + \frac{1}{2} k \epsilon^{ab} e_{ia} e_{jb} \right) \right\}. \quad (36)$$

We use the ansatz (20) to solve the equations of motions (18)-(19) by inserting $\Lambda = 0$ in them, and obtain the following solution:

$$\begin{aligned} M^2(r) &= \left(\frac{1}{D_1} \frac{dN(r)}{dr} \right)^2, & \eta_0(r) &= -\frac{kD_2}{D_1} N(r), & \eta_1(r) &= 0, \\ \eta_2(r) &= \frac{kD_2}{2D_1^2} N^2(r) + D_3, & \eta_3(r) &= D_2, \\ \omega(r) &= D_1 dt, & A(r) &= \left(\frac{k}{2D_1} N^2(r) + D_4 \right) dt + g(r) dr, \end{aligned} \quad (37)$$

where D_1, D_2, D_3 and D_4 are arbitrary constants, and $N(r), g(r)$ are arbitrary functions of r . The solution (37) describes a Ricci-flat space-time with zero scalar curvature ($\mathcal{R} = 0$). For $N^2(r) = 2D_1 r - D_5$, the metric in solution (37) reduces to the following Schwarzschild-type metric:

$$ds^2 = -(2D_1 r - D_5) dt^2 + \frac{dr^2}{2D_1 r - D_5}, \quad (38)$$

where D_5 is an arbitrary constant. The metric (38) has a coordinate singularity at

$$r = \frac{D_5}{2D_1}. \quad (39)$$

For $N^2(r) = (D_1 r - D_6)^2$, the metric in solution (37) turns out to be as follows:

$$ds^2 = -(D_1 r - D_6)^2 dt^2 + dr^2, \quad (40)$$

and has a coordinate singularity at

$$r = \frac{D_6}{D_1}, \quad (41)$$

where D_6 is an arbitrary constant. In section 4, we study the gauged Wess-Zumino-Witten model, and show that the latter metric solution (40) describes an exact (1+1)-dimensional Ricci-flat black hole in string theory.

3.3 Friedmann-Robertson-Walker (FRW) solutions

Cosmology of the two-dimensional Jackiw-Teitelboim gravity model is studied in Ref. [30]. Moreover, some cosmological solutions of the string inspired gravity coupled to the matter field both for dust-filled and radiation-filled space-times has been discussed in Ref. [31]. Here, to obtain some cosmological solutions for the equations of motions (18)-(19), we use Friedmann-Robertson-Walker metric as follows:

$$ds^2 = -dt^2 + a^2(t) \frac{dr^2}{1 - \kappa r^2}, \quad (42)$$

where $a(t)$ is the scale factor and describes the expansion of the world, and κ is a constant which can be equal to $-1, 0$ or $+1$ only. In $1+1$ dimensions, one can use the following coordinate transformation:

$$r \rightarrow \frac{1}{\sqrt{-\kappa}} \sinh(\sqrt{-\kappa} r) \quad \text{for} \quad \kappa < 0 \quad (43)$$

$$r \rightarrow \frac{1}{\sqrt{\kappa}} \sin(\sqrt{\kappa} r) \quad \text{for} \quad \kappa > 0 \quad (44)$$

to write the metric (42) as follows:

$$ds^2 = -dt^2 + a^2(t) dr^2. \quad (45)$$

Using the ansatz (45) for the metric in the equations (18)-(19) and after some calculations, one can obtain three different solutions corresponding to the negative, positive or zero values of Λ , as follows:

3.3.1 FRW Solution for $\Lambda < 0$

We have the following solution for the negative Λ :

$$\begin{aligned} a(t) &= \frac{\dot{a}(0)}{\sqrt{-\Lambda}} \sin(\sqrt{-\Lambda} t) + a(0) \cos(\sqrt{-\Lambda} t), \\ \omega(t, r) &= \frac{\Lambda}{k} \left(h(t, r) dt + s(t, r) dr \right), \quad A(t, r) = h(t, r) dt + \left(s(t, r) - \frac{k}{\sqrt{-\Lambda}} \xi_1(t) \right) dr, \\ \eta_0(r) &= \frac{dg(r)}{dr}, \quad \eta_1(t, r) = -\sqrt{-\Lambda} \xi_1(t) g(r), \\ \eta_2(t, r) &= -a(t) g(r), \quad \eta_3(t, r) = \frac{\Lambda}{k} a(t) g(r), \end{aligned} \quad (46)$$

where

$$\begin{aligned} \xi_1(t) &= a(0) \sin(\sqrt{-\Lambda} t) - \frac{\dot{a}(0)}{\sqrt{-\Lambda}} \cos(\sqrt{-\Lambda} t), \quad s(t, r) = \int dt \frac{\partial h(t, r)}{\partial r}, \\ g(r) &= C_1 \cosh(\sqrt{\hat{\Lambda}} r) - C_2 \sinh(\sqrt{\hat{\Lambda}} r), \quad \hat{\Lambda} = \dot{a}^2(0) - \Lambda a^2(0), \end{aligned} \quad (47)$$

C_1 and C_2 are constants, $h(t, r)$ is an arbitrary function of t and r , and dot denotes derivative with respect to the timelike coordinate t . $a(0)$ and $\dot{a}(0)$ in solution (46) are initial values of scale factor $a(t)$ and its time derivative $\dot{a}(t)$ at $t = 0$, respectively. Because the fields in solution (46) are functions of the radial coordinate r , this solution is not a homogenous solution. In order to obtain a homogenous solution, $h(t, r)$ must be r -independent $h(t, r) = h_0(t)$ and $C_1 = C_2 = 0$, where $h_0(t)$ is a function of timelike coordinate t only. Then, by this choice, all of the fields will be functions of the coordinate t only, and spatial homogeneity will be achieved. This solution will collapse at

$$t = \frac{1}{\sqrt{-\Lambda}} \arctan \left(-\sqrt{-\Lambda} \frac{a(0)}{\dot{a}(0)} \right). \quad (48)$$

The Hubble parameter $H(t)$ for this solution is as follows:

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \sqrt{-\Lambda} \left(\frac{\dot{a}(0) \cos(\sqrt{-\Lambda} t) - \sqrt{-\Lambda} a(0) \sin(\sqrt{-\Lambda} t)}{\dot{a}(0) \sin(\sqrt{-\Lambda} t) + \sqrt{-\Lambda} a(0) \cos(\sqrt{-\Lambda} t)} \right). \quad (49)$$

Using $\ddot{a}(t) = \Lambda a(t)$, the deceleration parameter $q(t)$ can be obtained as follows:

$$q(t) \equiv -\frac{a(t)\ddot{a}(t)}{\dot{a}^2(t)} = \left(\frac{\dot{a}(0) \sin(\sqrt{-\Lambda} t) + \sqrt{-\Lambda} a(0) \cos(\sqrt{-\Lambda} t)}{\dot{a}(0) \cos(\sqrt{-\Lambda} t) - \sqrt{-\Lambda} a(0) \sin(\sqrt{-\Lambda} t)} \right)^2, \quad (50)$$

which is obviously positive, and shows that the expansion of the universe is decelerating.

3.3.2 FRW Solution for $\Lambda > 0$

For positive Λ , one obtains the following solution:

$$\begin{aligned} a(t) &= \frac{\dot{a}(0)}{\sqrt{\Lambda}} \sinh(\sqrt{\Lambda} t) + a(0) \cosh(\sqrt{\Lambda} t), \\ \omega(t, r) &= \frac{\Lambda}{k} \left(\hat{h}(t, r) dt + \hat{s}(t, r) dr \right), & A(t, r) &= \hat{h}(t, r) dt + \left(\hat{s}(t, r) - \frac{k}{\sqrt{\Lambda}} \xi_2(t) \right) dr, \\ \eta_0(r) &= \frac{d\hat{g}(r)}{dr}, & \eta_1(t, r) &= \sqrt{\Lambda} \xi_2(t) \hat{g}(r), \\ \eta_2(t, r) &= -a(t) \hat{g}(r), & \eta_3(t, r) &= \frac{\Lambda}{k} a(t) \hat{g}(r), \end{aligned} \quad (51)$$

where

$$\begin{aligned} \hat{s}(t, r) &= \int dt \frac{\partial \hat{h}(t, r)}{\partial r}, & \xi_2(t) &= a(0) \sinh(\sqrt{\Lambda} t) + \frac{\dot{a}(0)}{\sqrt{\Lambda}} \cosh(\sqrt{\Lambda} t), \\ \hat{g}(r) &= \left\{ \begin{array}{ll} D_1 r + D_2 & \hat{\lambda} = 0 \\ D_1 \cosh(\sqrt{\hat{\lambda}} r) + D_2 \sinh(\sqrt{\hat{\lambda}} r) & \hat{\lambda} > 0 \\ D_1 \cos(\sqrt{-\hat{\lambda}} r) + D_2 \sin(\sqrt{-\hat{\lambda}} r) & \hat{\lambda} < 0 \end{array} \right\}, & \hat{\lambda} &= \dot{a}^2(0) - \Lambda a^2(0), \end{aligned} \quad (52)$$

D_1 and D_2 are constants, and $\hat{h}(t, r)$ is an arbitrary function. This solution is not homogenous, and as the previous solution, in order to have a homogenous solution we must put $\hat{h}(t, r) = \hat{h}_0(t)$ and $D_1 = D_2 = 0$. This solution will collapse for $|\frac{\dot{a}(0)}{a(0)}| \geq \sqrt{\Lambda}$, at

$$t = \frac{1}{\sqrt{\Lambda}} \operatorname{arctanh} \left(-\sqrt{\Lambda} \frac{a(0)}{\dot{a}(0)} \right), \quad (53)$$

but for $|\frac{\dot{a}(0)}{a(0)}| < \sqrt{\Lambda}$, it does not collapse. The Hubble parameter $H(t)$ for this solution is as follows:

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \sqrt{\Lambda} \left(\frac{\dot{a}(0) \cosh(\sqrt{\Lambda} t) + \sqrt{\Lambda} a(0) \sinh(\sqrt{\Lambda} t)}{\dot{a}(0) \sinh(\sqrt{\Lambda} t) + \sqrt{\Lambda} a(0) \cosh(\sqrt{\Lambda} t)} \right). \quad (54)$$

Using $\ddot{a}(t) = \Lambda a(t)$, the deceleration parameter $q(t)$ can be obtained as follows:

$$q(t) \equiv -\frac{a(t)\ddot{a}(t)}{\dot{a}^2(t)} = -\left(\frac{\dot{a}(0) \sinh(\sqrt{\Lambda} t) + \sqrt{\Lambda} a(0) \cosh(\sqrt{\Lambda} t)}{\dot{a}(0) \cosh(\sqrt{\Lambda} t) + \sqrt{\Lambda} a(0) \sinh(\sqrt{\Lambda} t)} \right)^2, \quad (55)$$

which is obviously negative. Note that for $\dot{a}(0) = \sqrt{\Lambda} a(0)$, the scale factor of the solution (51) has the following exponential form:

$$a(t) = a(0) e^{\sqrt{\Lambda} t}, \quad (56)$$

and leads to a constant Hubble parameter $H = \sqrt{\Lambda}$ and negative deceleration parameter $q = -1$, which means that the expansion of the universe is accelerating.

3.3.3 FRW Solution for $\Lambda = 0$

For $\Lambda = 0$, we obtain the following solution:

$$\begin{aligned}
a(t) &= \dot{a}(0)t + a(0), \\
\omega(t, r) &= \dot{a}(0)dr, \quad A(t, r) = \bar{h}(t, r)dt + \left\{ \bar{s}(t, r) - k\left(\frac{1}{2}\dot{a}(0)t^2 + a(0)t + E_3\right) \right\}dr, \\
\eta_0(r) &= -\frac{d\bar{g}(r)}{dr}, \quad \eta_1(t, r) = -\dot{a}(0)\bar{g}(r), \\
\eta_2(t, r) &= a(t)\bar{g}(r), \quad \eta_3(t, r) = 0,
\end{aligned} \tag{57}$$

where

$$\bar{s}(t, r) = \int dt \frac{\partial \bar{h}(t, r)}{\partial r}, \quad \bar{g}(r) = E_1 \cosh(\dot{a}(0)r) + E_2 \sinh(\dot{a}(0)r),$$

E_1 , E_2 and E_3 are arbitrary constants, and $\bar{h}(t, r)$ is an arbitrary function of t and r . To obtain a homogenous solution, $\bar{h}(t, r)$ must be indepenrent of coordinate r ($\bar{h}(t, r) = \bar{h}_0(t)$), and also we must have $E_1 = E_2 = 0$. This solution obviously will collapse at

$$t = -\frac{a(0)}{\dot{a}(0)}. \tag{58}$$

The Hubble parameter $H(t)$ and the deceleration parameter $q(t)$ for this solution can be obtained as follows:

$$H(t) = \frac{\dot{a}(0)}{\dot{a}(0)t + a(0)}, \quad q(t) = 0, \tag{59}$$

which show that expansion of the universe is without acceleration. In the next section, by studying the gauged Wess-Zumino-Witten model, we show that the cosmological metric solution (57) is also an exact solution to the string theory.

4 $[SL(2, \mathbb{R}) \times \mathbf{R}]/[U(1) \times U(1)]$ gauged Wess-Zumino-Witten model

In this section, we study G/H gauged Wess-Zumino-Witten model to obtain an exact solution to the string theory. Let the Lie group G is $G = SL(2, \mathbb{R}) \times \mathbf{R}$, and $H = U(1) \times U(1)$ is it's subgroup. If g is an element of the Lie group G , then the Wess-Zumino-Witten action can be written as follows [32]:

$$L(g) = \frac{k}{4\pi} \int_{\Sigma} d^2z \operatorname{Tr}(g^{-1} \partial g g^{-1} \bar{\partial} g) - \frac{k}{12\pi} \int_B \operatorname{Tr}(g^{-1} dg)^3, \tag{60}$$

where B is a three dimensional manifold and Σ is it's boundary. By introducing the gauge fields $\mathbf{A}, \bar{\mathbf{A}}$ which takes their values in the Lie algebra of H , the gauged Wess-Zumino-Witten action having the local axial symmetry $g \rightarrow hgh$, $h \in H$ can be written as follows:

$$L(g, \mathbf{A}) = L(g) + \frac{k}{2\pi} \int_{\Sigma} d^2z \operatorname{Tr}(\mathbf{A} \bar{\partial} g g^{-1} + \bar{\mathbf{A}} g^{-1} \partial g + \mathbf{A} \bar{\mathbf{A}} + \mathbf{A} g \bar{\mathbf{A}} g^{-1}), \tag{61}$$

where $u(1) \oplus u(1)$ Lie algebra valued gauge fields $\mathbf{A}, \bar{\mathbf{A}}$ have the following forms:

$$\mathbf{A} = A_1 \mathbf{u}_1 + A_2 \mathbf{u}_2, \quad \bar{\mathbf{A}} = \bar{A}_1 \mathbf{u}_1 + \bar{A}_2 \mathbf{u}_2, \tag{62}$$

where \mathbf{u}_1 and \mathbf{u}_2 are two nontrivial linear combinations of the generators X_1 and X_2 , corresponding to each of two $u(1)$ algebras, as follows:

$$\mathbf{u}_1 = \mathbf{X}_1 + \mathbf{X}_2, \quad \mathbf{u}_2 = \mathbf{X}_1 - \mathbf{X}_2. \tag{63}$$

We parameterize $G = SL(2, \mathbb{R}) \times \mathbf{R}$ group by the following group element: [6, 26]

$$g = \begin{pmatrix} a & u \\ -v & b \end{pmatrix} e^x, \quad ab + uv = 1. \tag{64}$$

Using the group element (64), one can rewrite the Wess-Zumino-Witten action (60) as follows:

$$L(g) = -\frac{k}{4\pi} \int_{\Sigma} d^2z (\partial u \bar{\partial} v + \partial v \bar{\partial} u + \partial a \bar{\partial} b + \partial b \bar{\partial} a) + \frac{k}{2\pi} \int_{\Sigma} d^2z \partial x \bar{\partial} x + \frac{k}{2\pi} \int_{\Sigma} d^2z (\partial a \bar{\partial} b - \partial b \bar{\partial} a) \log(u). \quad (65)$$

Then, we gauge two-dimensional subgroup $H = U(1) \times U(1)$ generated by

$$\mathbf{X}_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{X}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (66)$$

Using the group element (64), the following gauge fixing choices

$$a = b, \quad u = v, \quad (67)$$

and by eliminating the gauge fields A_i, \bar{A}_i , with $i = 1, 2$ (by use of the variations of the action with respect to them), the gauged WZW action (61) can be rewritten in the following form:

$$L(g, \mathbf{A}) = \frac{k}{2\pi} \int_{\Sigma} d^2z \left(\frac{\partial u \bar{\partial} u}{3 + u^2} + \partial x \bar{\partial} x \right). \quad (68)$$

Using the following field redefinition:

$$u(t, r) = \sqrt{3} \sinh\left(\frac{1}{2\alpha}(\alpha r + \beta) \sinh(\alpha t)\right), \quad x(t, r) = \frac{i}{2\alpha}(\alpha r + \beta) \cosh(\alpha t), \quad (69)$$

the gauged WZW action (68) becomes

$$L(g, \mathbf{A}) = -\frac{k}{8\pi} \int_{\Sigma} d^2z \left(-(\alpha r + \beta)^2 \partial t \bar{\partial} t + \partial r \bar{\partial} r \right), \quad (70)$$

where α and β are arbitrary constants. Then, the obtained gauged WZW action (70) describes a string propagating in a space-time with the following metric

$$ds^2 = -(\alpha r + \beta)^2 dt^2 + dr^2, \quad (71)$$

which is an exact Ricci-flat black hole in string theory. By assuming $\alpha = D_1$ and $\beta = -D_6$, the latter metric solution (71) is precisely same as the metric (40) obtained as a solution of our gravity model (9). One-loop beta function equations in 1 + 1 dimensions have the following forms [33]:

$$R_{\mu\nu} + 2\nabla_{\mu} \nabla_{\nu} \phi = 0, \quad (72)$$

$$R + \frac{8}{k} + 4\nabla^2 \phi - 4(\nabla \phi)^2 = 0, \quad (73)$$

where R and $R_{\mu\nu}$ are the scalar curvature and Ricci tensor of the target space, ϕ is the dilaton field, and $\frac{8}{k}$ is the cosmological constant term. By requiring that the metric (71) must obey the one-loop beta function equations (72) and (73), we obtain the following relation for the dilaton field using (72):

$$\phi(t, r) = \frac{(\alpha r + \beta)}{\alpha} (c_1 \sinh(\alpha t) + c_2 \cosh(\alpha t)), \quad (74)$$

where c_1 and c_2 are some constants. By substituting $\phi(t, r)$ in (73), one can obtain the following relation between the constants c_1 and c_2 :

$$c_2^2 - c_1^2 = \frac{2}{k}. \quad (75)$$

In the same way, using another field redefinition as follows:

$$u(t, r) = \sqrt{3} \sinh\left(\frac{1}{2\hat{\alpha}}(\hat{\alpha} t + \hat{\beta}) \sinh(\hat{\alpha} r)\right), \quad x(t, r) = \frac{i}{2\hat{\alpha}}(\hat{\alpha} t + \hat{\beta}) \cosh(\hat{\alpha} r), \quad (76)$$

the gauged WZW action (68) turns out to have the following form:

$$L(g, \mathbf{A}) = \frac{k}{8\pi} \int_{\Sigma} d^2z \left(-\partial t \bar{\partial} t + (\hat{\alpha} t + \hat{\beta})^2 \partial r \bar{\partial} r \right), \quad (77)$$

which describes a string propagating in a space-time with the following cosmological metric:

$$ds^2 = -dt^2 + (\hat{\alpha} t + \hat{\beta})^2 dr^2, \quad (78)$$

where $\hat{\alpha}$ and $\hat{\beta}$ are arbitrary constants. By assuming $\hat{\alpha} = \dot{a}(0)$ and $\hat{\beta} = a(0)$, this metric is precisely same as the FRW metric (57) which is obtained in solving the equations of motion for our gravity model discussed in previous section. Using (72) and (73), one can obtain the following relation for the dilaton field:

$$\phi(t, r) = \frac{(\hat{\alpha} t + \hat{\beta})}{\hat{\alpha}} (d_1 \sinh(\hat{\alpha} r) + d_2 \cosh(\hat{\alpha} r)), \quad d_1^2 - d_2^2 = \frac{2}{k'}, \quad (79)$$

where d_1 and d_2 are constants. Note that the black hole metric (71) converts to the FRW metric (78), and vice versa, using the following coordinate transformation:

$$t \rightarrow \hat{r}, \quad r \rightarrow \hat{t}. \quad (80)$$

5 Conclusions

We have presented a four-dimensional extension of the Poincaré algebra in $(1+1)$ -dimensional space-time. Using this algebra, we have constructed a gauge theory of gravity which is dual (canonically transformed) to the *AdS* gauge theory of gravity, under special conditions. We have also obtained black hole and Friedmann-Robertson-Walker (FRW) cosmological solutions for this model. Then, using $[SL(2, \mathbb{R}) \times \mathbf{R}]/[U(1) \times U(1)]$ gauged Wess-Zumino-Witten action, we have shown that some of the black hole and cosmological solutions of our gravity model are exact $(1+1)$ -dimensional solutions for string theory.

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